Code: 20BS1101

I B.Tech - I Semester – Regular / Supplementary Examinations – APRIL 2022

CALCULUS AND LINEAR ALGEBRA

(Common for All Branches)

Duration: 3 hours Max. Marks: 70

Note: 1. This paper contains questions from 5 units of Syllabus. Each unit carries 14 marks and have an internal choice of Questions.
2. All parts of Question must be answered in one place.

UNIT – I

1. a) By applying elementary row operations, find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. 7 M

b) Investigate for what values of λ and μ the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have i) no solution ii) a unique solution iii) an infinite number of solutions. 7 M

OR

2. a) Construct two non-singular matrices P and Q such that PAQ is in normal form for the matrix 7 M

$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 2 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$. Hence find its rank.

b) Determine the values of k for which the following equations has non-trivial solutions and find them. 7 M

$(k-1)x + (4k-2)y + (k+3)z = 0$,

$(k-1)x + (3k+1)y + 2kz = 0$,

$2x + (3k+1)y + 3(k-1)z = 0$.

UNIT – II

3. a) Verify Cayley Hamilton theorem for the matrix 7 M

$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Hence find A^{-1}.

b) Find the eigen values and eigen vectors of the matrix 7 M

$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

OR

4. a) By applying Cayley - Hamilton theorem, express 7 M

$2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A,

where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

b) Examine the nature of the quadratic form 7 M

$3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$.

UNIT-III

5. a) By applying Lagrange’s mean value theorem, prove 7 M

that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\left(\frac{3}{5}\right) > \frac{\pi}{3} - \frac{1}{8}$.

b) Verify Rolles's theorem for $f(x) = (x-a)^m (x-b)^n$ in $[a, b]$, where m, n are positive integers. 7 M

OR

6. a) Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$. Correct to 4 decimal places. 7 M

b) Verify the Cauchy's mean value theorem for the functions $f(x) = \log_e x$ and $g(x) = \frac{1}{x}$ in the interval $[1, e]$. 7 M

UNIT – IV

7. a) If $x = r\cos\theta$, $y = r\sin\theta$ then show that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$. 7 M

b) Prove that the functions $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ are functionally dependent and also find the relation between them. 7 M

OR

8. a) Discuss the maxima and minima of $f(x,y) = x^3 y^2 (1 - x - y)$. 7 M

b) Find the point on the plane $x + 2y + 3z = 4$ that is closest to the origin. 7 M

UNIT – V

9. a) Change the order of integration and evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx.$ 7 M

b) Evaluate $\iint r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$. 7 M

OR

10.

a) Evaluate the triple integral $\int_0^1\int_y^1\int_0^{1-x} xdzdxdy$. 7 M

b) By applying triple integration, find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. 7 M